An Adventure into Outer Space: Stockhausen's *Lichter—Wasser* and the Analysis of Spatialized Music

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The movement of sound in the space around listeners—or, musical "spatialization"—is one of the oldest techniques for enlivening the experience of music. Many musicians and composers throughout history have found ingenious ways of using physical space, ranging from singers of medieval chant to composers such as Gabrieli, Dowland, Berlioz, Mahler, and Ives.[1] However, composers' abilities to control space in their music increased dramatically in the 1950s with the development and use of multi-channel electronic sound projection.[2] Within the last twenty years or so, a second revolution has occurred thanks to sophisticated computer software and increased processing power. These innovations have allowed designers and composers unprecedented control in creating complex effects in the space around listeners, and also spurred composers to develop new compositional techniques to organize this musical domain.[3] Electronic spatialization has also influenced non-electronic instrumental music.

Several recent studies have appreciated the possibilities for analyzing the compositional techniques that composers have developed to spatialize music.[4] Some of the most useful work in this regard has been directed towards music of Xenakis, who composed several notable spatialized pieces including *Pithoprakta* (1956), *Eonta* (1963–64), *Terretektorh* (1965–66), and others (Maria Harley 1994, 292). Many other studies focus on the spatialized music of Stockhausen. For the analyst, this musical spatialization poses many thorny problems. First, room acoustics play a significant role in the physical projection of sound energy and the ability of listeners to localize it. Second, humans react very differently to spatialized sound depending on their head and ear shape. Even though the eardrums produce the most significant acoustical input signals for the human auditory system, the external parts of the ear also play an important role (Blauert, 289–95). This generally causes a great deal more variation among listeners' abilities to localize spatial sound events than, say, their ability to determine pitch intervals or relative rhythmic durations under similar circumstances. Third, the localization of space through sound is affected by other musical parameters—for example, space perception can be altered by slight pitch variations mimicking the Doppler effect.[5] This means that if music perception is to play a role in analysis, several musical parameters must be taken into consideration, making the task more complex.[6] Finally, there are few if any existing historical methods or traditions that can help us to approach spatialization in an informed way.[7] There is not even a generally accepted way of measuring either real or apparent movement in spatial music—instead, it is usually written about in terms of the setup of loudspeakers, or the arrangement of musicians in a perfor-mance space.[8] An analogous situation in the pitch world might be somewhat akin to having only a sense of melodic contour, without the ability to reference the specific pitches involved.

Despite these challenges, it has become more and more difficult to ignore the spatial aspect of much contemporary music. Composers like Xenakis, Stockhausen, and many others have created complex and intricate spatial structures in their music which warrant investigation at a detailed level of analysis. Compositional work has gone on both in instrumental and electronic spatialization at places ranging from large laboratories like IRCAM, to suburban homes where teenagers' off-the-shelf desktop computers run software such as MAX/MSP or

Supercollider. Like many other analytical projects, investigating the structures present in spatial music can verify claims that a composer made, suggest ways of listening so that a work's more subtle aspects can be appreciated, and perhaps even inspire composers to find new ways of composing in space.

Roger Reynolds wrote in the pages of this journal over thirty years ago that "The equipment and much of the perceptual information that would allow an orderly examination of the geometry of sound already exists; what is lacking is informed strategy" (Reynolds 1978, 183). Following Reynolds' suggestion, I will develop an informed strategy for examining the geometry of sound in a spectacular work of spatial music—Stockhausen's *Lichter—Wasser* (1999–2000). In *Lichter—Wasser*, the movement of melodic ideas in an instrumental ensemble is created through a "discrete" technique (a term coined by Maria Harley), which hearkens back to the Venetian polychoral school of the Renaissance—or even earlier (Maria Harley 1994, 292).[9] In Stockhausen's piece, the *apparent* movement of melodic ideas in a stationary ensemble (as opposed to the *actual* movement of sounds through space through real physical movement) is the focus of a great deal of compositional design—perhaps, *the* central concern in its design. As we shall see, *Lichter—Wasser* provides fertile soil for developing analytical methodologies for certain types of spatial music, by employing relatively simple measuring techniques and mathematical graph theory.[10] The analysis will show that it is possible to measure spatial activity in Stockhausen's work, and then to analyze the measurements in various ways, offering a glimpse into the structure of physical space in a complex musical composition. This will not only permit verification of the composer's own claims, but also will enable harvesting a substantial yield of information about the piece, lending insight which is not able to be appreciated in any other way. One by-product will be the unearthing of a subtle, hidden aspect of spatial structure which elegantly reinforces the work's program.

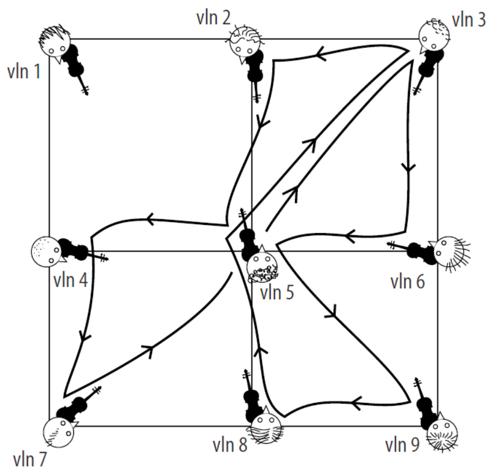
–1–

Before attempting to develop a methodology for a piece like *Lichter—Wasser*, we should consider a crucial premise. Should the focus be on the perceptions of listeners, or the data written in a score? The approach here will be the latter. Since perception of spatial music can vary widely depending on a multitude of factors, studying listeners' reactions could yield such a wide range of data that it might be difficult to make any general conclusions. It might also be impractical to set up an experiment for a work like *Lichter—Wasser*, since twenty-nine live musicians plus a synthesizer player and a conductor would be required for any tests.[11] According to James Harley, the "kinetic aspect of the sound" in Xenakis's *Terretektorh* (1965–66)—a work which is similar to *Lichter—Wasser* in certain fundamental ways—"is to some extent imaginary, as the composer conceives of the listening experience being different for each listener as if it were possible to move from one position to another" (James Harley 2004, 46).[12] Although a perceptual study might help to define a range of individual listening experiences, the advantage to basing the analysis on elements from a musical score is that—provided that it is clear about where sounds are and how they move—we can all more or less agree on a set of idealized spatial phenomena to analyze.

To help conceive of an analytical methodology, imagine how spatial motion could be measured by considering an all-instrumental ensemble with musicians scattered around the performance space. The apparent movement of a spatialized melody could be notated in the way indicated in Example 1. Arguably, the notes in this melody are connected together as it moves around the room because of the tune's familiarity. This is by no means the only way of creating musical spatialization, but it is one method that many composers have employed. In this example, nine violinists are scattered throughout the performance space. Looking down on this scene, we might see the arrangement shown in Example 2. After one musician plays a note or group of notes, the melody is passed on to another performer.[13]

How might the spatial motion be analyzed in this situation? It is not difficult to measure the size of the performance space and thereby derive the distance between each musician. Also, since the tempo is marked (♩ = 60), it is easy to calculate the approximate amount of time the melody stays at each musician. Since speed is a function of distance and time, it can easily be determined from this data. Continuing along these lines, the density of musical events in each part of the hall could be found by tallying up the number of times each musician plays. Clearly, violin 5 gets used the most, and poor violin 1 is left out of the game entirely—perhaps he is hired simply for his good looks. It is not difficult to make these basic measurements, yet the information we gather from them can tell a great deal about how space is used in the composition.

EXAMPLE 1: A SPATIALIZED MELODY

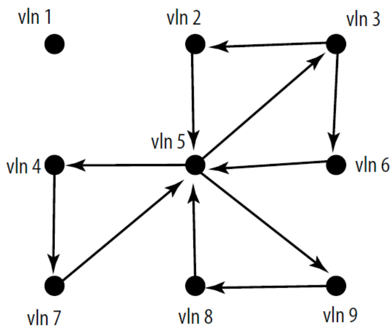


EXAMPLE 2: VISUAL REPRESENTATION OF SPATIAL MOVEMENT

What about the connections that are formed among musicians in their melodic "handoffs"? In the hypothetical spatial piece, fifty percent of the handoffs from violin 5 to another player are to violin 3. Violin 5 hands the melody to violins 4 and 9 twenty-five percent of the time. These simple observations can have an important bearing on the individual spatial characteristics of each instrumentalist in a piece. Further observations of the behavior of instrumentalists suggest that some function in very different ways than others. For example, violin 2 seems to act as a kind of "conduit," always receiving motion from one source (violin 3) and then always passing it to a single recipient (violin 5). Perhaps because of its central spatial location, violin 5 functions in a much different way. It receives motion from a variety of sources (violins 2, 6, 7, and 8) and then sends music out to many other recipients (violins 3, 4, and 9). The function of violin 5 might best be described as a kind of "hub" or "reflector." By measuring spatial motion, we can differentiate among several spatial functions which allow us to assign structural roles to instruments in a piece of spatial music. In this test, it is safe to say that different types of activity take place at different spatial locations.[14]

The spatial motion in the test piece can also be analyzed by representing the composite out-of-time structure of all spatial moves as a *mathematical graph*.[15] In general, a graph is a set of vertices (usually called "nodes") and a set of edges that connect those vertices.[16] Violinists can be imagined as vertices, and their melodic handoffs as edges in the graph. Example 3 shows a graph represen-tation of the piece's spatial moves. Since there are arrows on the graph, this particular representation is a *directed graph*. This graph is a more abstract structure which tells us what connections the composer has made between instruments in the short piece, without reference to the number of times that particular connection is made.[17]

An *adjacency matrix* is a convenient way of representing the graph structure, so that computations may be made on it. Adjacency matrix **A** contains a **0** entry if there is no edge between two vertices, and a **1** if there is an edge.[18] Such a matrix can be seen in Example 4. Representing the spatial moves in this way allows calculation of the shapes and paths that might be possible through the graph, given the connections that the composer has made. Multiplying the **A** matrix by itself, using standard matrix multiplication, creates another matrix called the $\mathbf{A}^2$ matrix. Given the connections or links that the composer has made,



EXAMPLE 3: REPRESENTATION OF SPATIAL STRUCTURE AS A GRAPH

A Matrix	Vln 1	Vln 2	Vln 3	Vln 4	Vln 5	Vln 6	Vln 7	Vln 8	Vln 9
Vln 1	0	0	0	0	0	0	0	0	0
Vln 2	0	0	0	0	1	0	0	0	0
Vln 3	0	1	0	0	0	1	0	0	0
Vln 4	0	0	0	0	0	0	1	0	0
Vln 5	0	0	1*	1	0	0	0	0	1
Vln6	0	0	0	0	1	0	0	0	0
Vln 7	0	0	0	0	1	0	0	0	0
Vln 8	0	0	0	0	1	0	0	0	0
Vln 9	0	0	0	0	0	0	0	1	0

A^2 Matrix	Vln 1	Vln 2	Vln 3	Vln 4	Vln 5	Vln 6	Vln 7	Vln 8	Vln 9
Vln 1	0	0	0	0	0	0	0	0	0
Vln 2	0	0	1	1	0	0	0	0	1
Vln 3	0	0	0	0	2	0	0	0	0
Vln 4	0	0	0	0	1	0	0	0	0
Vln 5	0	1	0*	0	0	1	1	1	0
Vln6	0	0	1	1	0	0	0	0	1
Vln 7	0	0	1	1	0	0	0	0	1
Vln 8	0	0	1	1	0	0	0	0	1
Vln 9	0	0	0	0	1	0	0	0	0

*Since this is a directed graph, the A^2 matrix correctly indicates that there is no way to get from vln 5 to vln 3 involving two moves, but the A matrix shows that there is one way of getting there in one move.

EXAMPLE 4: MATRIX REPRESENTATIONS OF SPATIAL STRUCTURE

the $\mathbf{A^2}$ matrix indicates the number of paths that could connect a maximum number of *three* musicians together.[19] This makes it possible to determine what longer paths or shapes are possible in a piece of spatial music, given the composer's own choices.

Our $\mathbf{A^2}$ matrix confirms that there are two different paths involving three violinists that the melody follows to get from violin 3 to violin 5.[20] A *path* in this context is simply a set of points in space that is traversed during the course of the composition. In other words, a path through space involving a finite number of vertices is the spatial cousin of the familiar pitch-class set. Of course, paths can be related using the mathematical transformations of rotation, reflection, and translation. It is easy to see that several sets of shapes in the hypothetical piece are actually related in this way. In music which is spatialized like this, adjacency matrices can make a deeper structure visible.

A second technique that can be used to analyze the graph relates to the idea of centrality. Although there are many ways to measure centrality in graph theory, all of the various techniques are used to determine how important a node is in a graph. In general, a node is more central in a graph if it has more directed edges coming into it (*inflow*) and going out from it (*outflow*). In the test piece, it is easy to determine by observation alone that violin 5 is the most central node in our graph, since it has the most inflow (four edges) and the most outflow (three edges). The concept of centrality will be explored in greater detail shortly.

Abstracting the links among instrumentalists in spatial music as a mathematical graph helps to calculate a set of expectations, which could potentially add richness to the listening experience. Although the composer's choice about where to place musicians in a spatial piece already indicates a certain amount of compositional planning, analyzing the spatial moves around the space allows a deeper level of compositional design to come to light. Representing spatial moves as a graph permits the use of many mathematical tools developed to evaluate graph structures. In the next section, the basic techniques of analysis developed so far will be applied to a real-world situation—Stockhausen's *Lichter—Wasser*.

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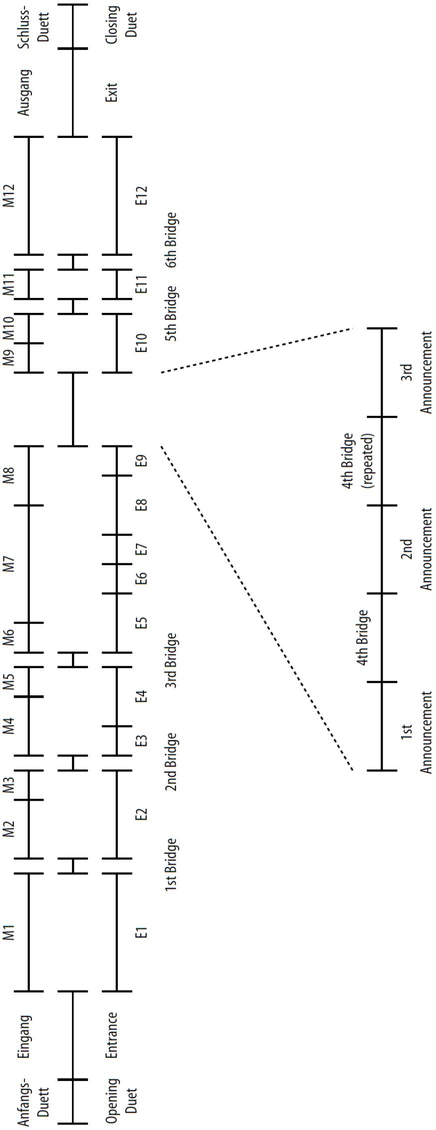
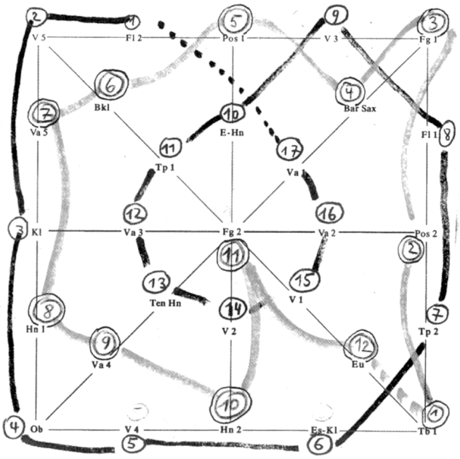
Although the elaborate composing of spatial movements is not uncommon in Stockhausen's music, the *Sunday* opera—the last-composed opera in his epic *Licht* cycle—is particularly rich in its use of spatialization.[21] Stockhausen imagined the entire *Sunday* opera as the day of "mystical union" between Michael and Eve.[22] There is very little dramatic action in *Sunday*. Instead, the opera's six scenes have a pronounced ritualistic and meditative character. Each scene also has a distinctive spatial imprint and most include elaborate spatial activity. The opening scene, *Lichter—Wasser*, is the focus of our analysis below, and includes an elaborate entrance and exit procession where musicians ambulate to their assigned locations in the performance space. The spatial motion is real in this introductory section, but in the instrumental music of the rest of the work, the spatial motion is *apparent*: in other words, melodies are passed from place to place among musicians who are situated in fixed locations. *Engel-Prozessionen*, scored for *a capella* choir, follows as scene two. The choir, which is divided into seven smaller ensembles, processes around the hall in groups while singing in seven different languages. The path of each choir group through the hall is carefully indicated in the preface to the score. Unlike *Lichter—Wasser*, *Engel-Prozessionen* features real physical spatial movement through the performance area. In the third scene, *Licht-Bilder*, four instrumentalists on stage move around in stylized patterns, all indicated in the score. Scene four—*Düfte-Zeichen*—is a work for seven singers, boy's voice, and synthesizer. Solos, duets, or trios are sung while different types of incense are burned. The music in *Düfte-Zeichen* takes place on seven podia, set up in an arc in front of the audience. *Hoch-Zeiten*, which is scene five, calls for five-part choir and five-part orchestra—but, these two ensembles are located in entirely different halls, and mixed together electronically so they are occasionally "blended" for listeners in each of the respective performance sites. The final part of the *Sunday* opera, *Sonntags-Abschied*, is not a scene, strictly speaking: rather, it is a wordless transcription of *Hoch-Zeiten* for five synthesizer players, whose sounds are spatialized around the audience in a loudspeaker array. Like Stockhausen's other "Farewells," this can be performed outside as the audience leaves the concert hall or opera house.

The first scene of Stockhausen's *Sunday* opera, called *Lichter—Wasser*, or "Lights—Waters," (1999–2000) has an elaborate spatial structure. According to Stockhausen, "life depends completely on light and water"; for him, they are the essential ingredients for life in the universe.[23] *Lichter—Wasser* is a kind of ritual celebration of the planets, moons, and other celestial bodies in our local solar system. The overall formal structure of the work, as can be seen in Example 5, consists of a series of alternating "wave" and "bridge" sections framed by an "entrance" (*Eingang*), an "exit" (*Ausgang*), and brief duets. The twelve wave sections do not always line up, but when they do, a "bridge" section occurs.[24] Apparent spatial motion in these bridge sections is less active than in the wave sections, which contain the most intricate spatial movement.

Lichter—Wasser is scored for twenty-nine instrumentalists, two singers, and a synthesizer player. The instrumentalists are arranged in a geometric pattern around the audience, as shown in Example 6. They are divided into two orchestras. Each orchestra contains a variety of



EXAMPLE 5: FORMAL STRUCTURE, *LICHTER—WASSER*
(WAVE SECTIONS ARE ABBREVIATED M1, E1, ETC.)

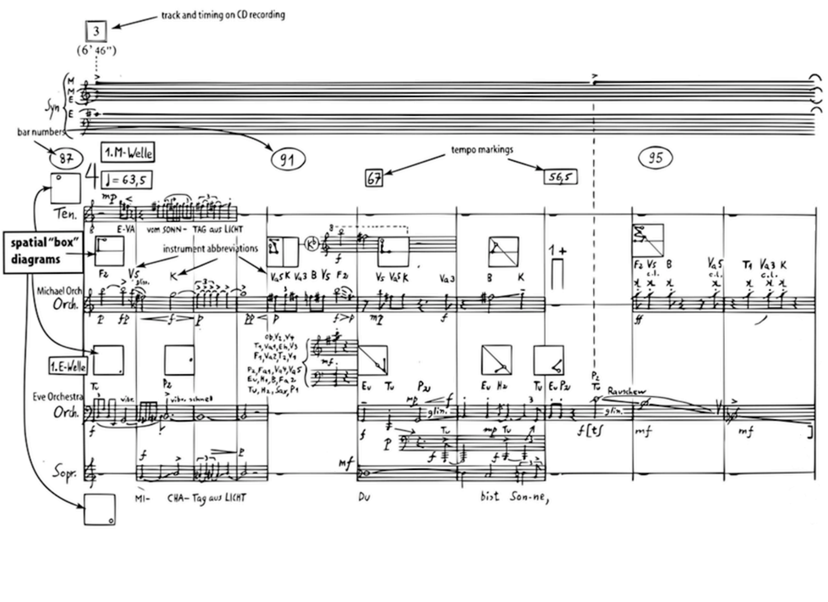


EXAMPLE 6: SPATIAL ARRANGEMENT OF 29 INSTRUMENTALISTS IN *LICHTER—WASSER*. "EVE" INSTRUMENTS ARE IN LIGHT GREY; "MICHAEL" INSTRUMENTS ARE IN DARK GREY.

instruments. The Michael orchestra contains far more string instruments—eight—as opposed to the Eve orchestra, which only contains two. The light-grey line (originally, green) connects the instruments in the Eve orchestra, while the darker line (originally, blue) indicates the Michael instruments. The lines show what I call "basic cycles" of movement for each orchestra.[25] Chairs for the audience are set up in between the instrumentalists. The vocalists—one representing Eve and the other Michael—sing while ambulating around the audience in a set pattern. Stockhausen originally conceived that "the rotations of the notes in space are related to the rotations of the nine planets and 61 moons of our solar system, whose names, astronomical characteristics and significances are sung" (Stockhausen 2001, 5).[26] Stockhausen later abandoned the attempt to base the spatial motion of his music directly on the motions of actual heavenly bodies, but still put a great emphasis on giving listeners the chance to hear the spatial motion around them.[27]

Although there is quite a lot of compositional complexity in the motions of the vocalists in *Lichter—Wasser*, the great majority of apparent melodic spatial movement occurs in the instrumental ensemble during the twelve wave sections.[28] In fact, one might think of the twelve waves as a series of two-part "spatial inventions," since counterpointed Michael and Eve spatial melodies—which will be explored further below—continually weave their way around space in elaborate patterns. Each of the two groups of instrumentalists plays a melody associated with their character, Michael or Eve, during the wave sections. However, musicians play only a note or group of notes from their melody, and then pass it on to a nearby instrumentalist. The situation is essentially the same as the one described earlier with the nine violinists. As can be seen in Example 7, Stockhausen indicated the sequence of spatial moves in the score by abbreviating each instrument above the melodic chunk it is supposed to play, instead of writing out a much more cumbersome conventional score. Occasional "box diagrams" indicate the shape of spatial movement as it would appear looking down on the scene from above, and aid the conductor in coordinating the moves among players.[29]

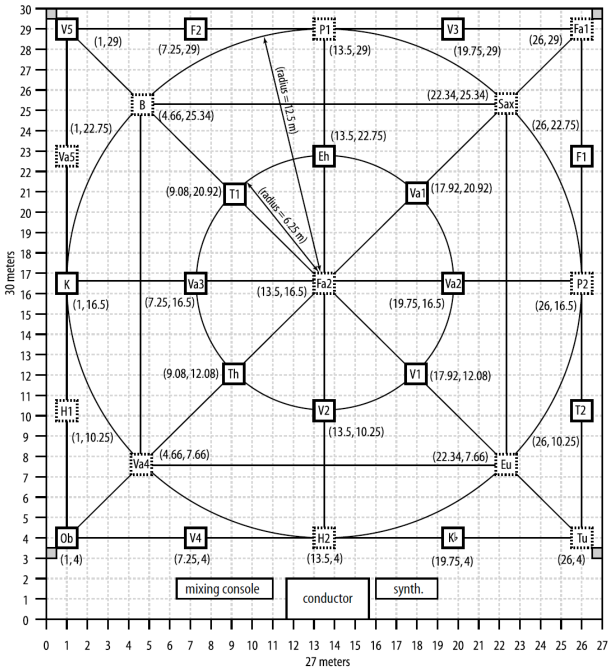
What melodic ideas are played in the Michael and Eve orchestras? It is well known that Stockhausen used a three-part formula "super-formula" to organize the structure of his seven *Licht* operas. The



EXAMPLE 7: BEGINNING OF WAVE 1 IN *LICHTER—WASSER*.

techniques of composing with a formula are generally well-understood, and are discussed at length in Kohl 1990, Coenen 1994, Stockhausen 1998, 12–28, Bandur 1999, Schwerdtfeger 1999, Maconie 2005, Toop 2006, and several other published books and articles. In *Licht*, the three formulas (one each for Eve, Michael, and Lucifer) began as "nuclear formulas" containing 12, 13, and 11 notes respectively. These nuclear formulas are similar to conventional tone rows, except that the Michael formula has one extra note at the end—a D—which is *both* the first note of the Michael formula, *and* the one note that is missing from the Lucifer nuclear formula.[30] Through the addition of various "Akzidenzien" (including "echo," "scale," and "wind"), pauses, and so-called "colored pauses," the three nuclear formulas acquire another layer of characteristic qualities, and become a "formula."[31] In one way or another—as background, middleground, or foreground material—these formulas are present throughout the seven opera-cycle *Licht*.[32] By expanding, or "projecting" the formulas over a larger span of time, the work acquires a kind of unity. *Lichter—Wasser* itself is based on a small segment of the superformula near its end—what Stockhausen called the "Sunday limb." On a more foreground level, the Michael and Eve formulas form the structural melodic material for each of the twelve wave sections in *Lichter—Wasser*, though with a few modifications.[33] Although the superformula melodies may not be as familiar to listeners as, for instance, the tune cited in Example 1, it is still possible to hear one of these "main tunes" running through the texture. It is more likely that the listener will be able to follow these melodies because their contour is relatively stable: it would be hard to imagine the Michael-formula starting with an ascending fifth instead of a descending fourth.[34] This makes it easier to follow the musical thread as it whirls around one's head.[35]

By laying a Cartesian coordinate system over the *Lichter—Wasser* space, as shown in Example 8, we can estimate the coordinates of each musician to within about a half-meter accuracy.[36] Then, it is easy to derive the distance between each instrumentalist. The technique of using a coordinate system was first suggested by Hofmann, who uses a kind of "tin-can" diagram in polar coordinates to show certain aspects of movement in Xenakis's music (Hofmann 2008, 85, 99). After determining the location of each performer in Cartesian coordinates, one can calculate the length of time each musician plays a chunk of melody. This is relatively easy to do, since Stockhausen notated tempo changes throughout, and the score is extremely clear in this regard.[37] Knowing the location of each musician and the length of time it takes to move between them makes it possible to calculate the rate of movement for each segment of the work. For the analysis, the roughly



Coordinates of Each Instrument

instrument	abbreviation	x-coord.	y-coord.
violin 5	V5	1.00	29.00
flute 2	F2	7.25	29.00
trombone 1	P1	13.50	29.00
violin 3	V3	19.75	29.00
bassoon 1	Fa1	26.00	29.00
bass clarinet	B	4.66	25.34
saxophone	Sax	22.34	25.34
viola 5	Va5	1.00	22.75
english horn	Eh	13.50	22.75
flute 1	F1	26.00	22.75
trumpet 1	T1	9.08	20.92
viola 1	Va1	17.92	20.92
clarinet	K	1.00	16.50
viola 3	Va3	7.25	16.50
bassoon 2	Fa2	13.50	16.50
viola 2	Va2	19.75	16.50
trombone 2	P2	26.00	16.50
tenor horn	Th	9.08	12.08
violin 1	V1	17.92	12.08
horn 1	H1	1.00	10.25
violin 2	V2	13.50	10.25
trumpet 2	T2	26.00	10.25
viola 4	Va4	4.66	7.66
euphonium	Eu	22.34	7.66
oboe	Ob	1.00	4.00
violin 4	V4	7.25	4.00
horn 2	H2	13.50	4.00
e-flat clarinet	K♭	19.75	4.00
tuba	Tu	26.00	4.00

Key

= Instrument in Michael Orchestra

= Instrument in Eve Orchestra

= Location of loudspeaker

EXAMPLE 8: OVERLAY OF CARTESIAN COORDINATE SYSTEM OF THE INSTRUMENTALISTS IN *LICHTER—WASSER*. (THE HALL IN WHICH THE PREMIERE TOOK PLACE MEASURED 27×30 METERS.)

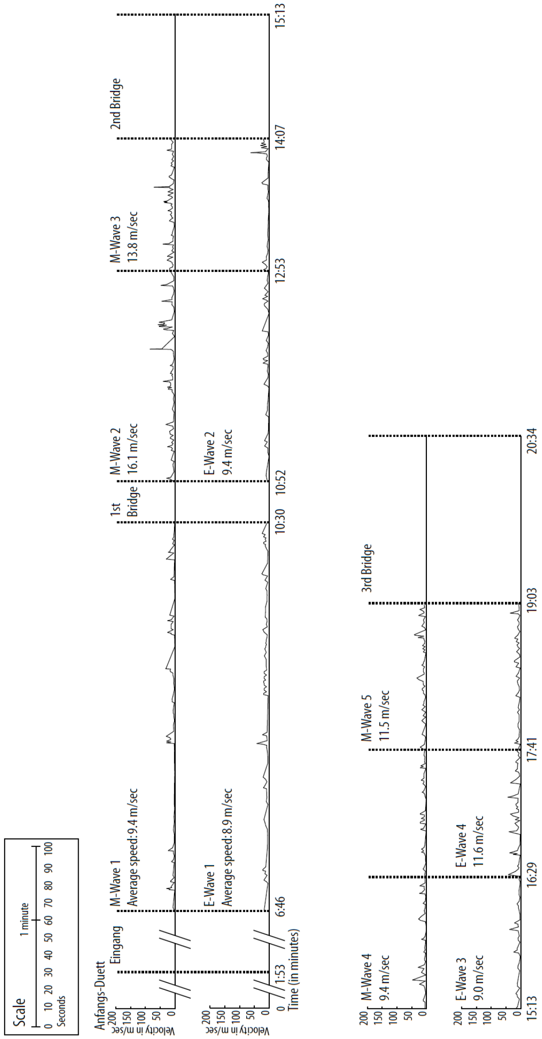
3,800 spatial moves were sequenced along with their precise timing, and then the data were stored in a database.[38] Many of the most significant results of this analysis are shown in Example 9.

The large graph in Example 10 plots some aspects of the analysis that were used to generate Example 9. Stockhausen claimed in his lectures on *Lichter—Wasser* that the rate of spatial movement increased from one wave structure to the next. The data shown in Example 10

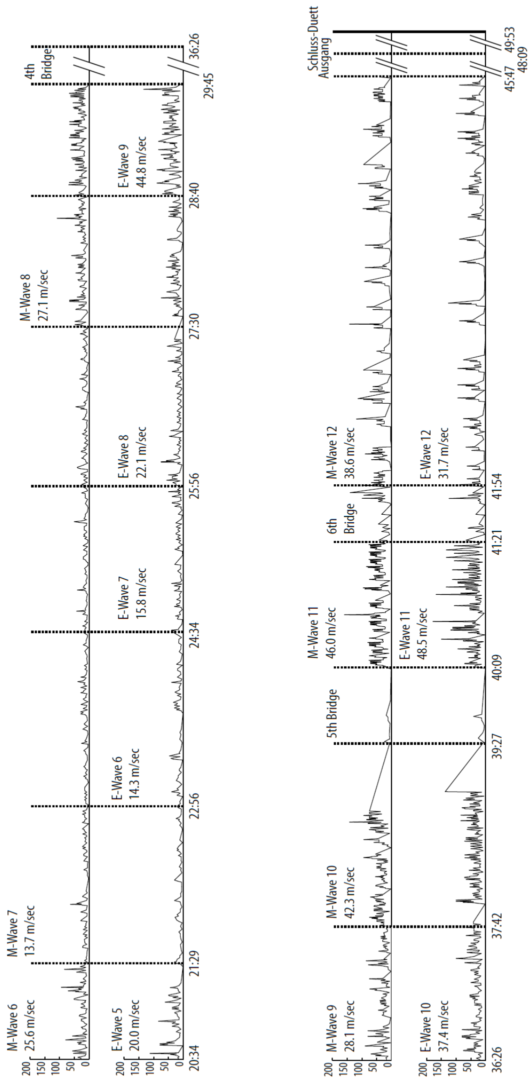
	Total Number of Moves	Average Length of a Move (in meters)	Average Duration of a Move (in seconds)	Average Velocity (in m/sec)	Total Distance Traversed (in meters)	Calculated Duration (in seconds)	Duration on CD Recording (in seconds)*
M1	105	7.68	2.07	9.42	806.01	217.22	224
E1	100	10.85	2.18	8.89	1084.59	217.60	224
M2	105	8.07	0.99	16.12	847.60	104.13	121
E2	104	9.44	1.59	9.44	982.14	165.72	195
M3	84	7.21	0.72	13.84	605.64	60.43	74
E3	43	10.61	1.60	8.97	456.19	68.95	76
M4	115	6.84	1.14	9.40	786.62	131.30	148
E4	116	9.84	1.14	11.60	1141.36	132.46	154
M5	68	8.29	1.00	11.53	555.39	67.25	82
E5	134	11.09	0.88	20.00	1474.80	117.06	142
M6	91	8.36	0.44	25.64	752.79	39.97	55
E6	83	10.08	0.97	14.31	826.74	76.56	98
M7	401	7.85	0.75	13.70	3148.32	299.45	361
E7	98	8.51	0.76	15.77	833.77	74.27	82
M8	254	8.02	0.39	27.06	2036.04	99.53	135
E8	213	8.50	0.58	22.10	1810.58	123.36	164
M9	139	8.39	0.43	28.07	1166.22	59.71	76
E9	123	12.21	0.37	44.76	1501.23	45.68	65
M10	155	9.59	0.34	42.31	1485.80	52.93	105
E10	291	10.20	0.38	37.38	2968.69	110.46	181
5th Bridge (M)	11	11.73	2.45	9.30	129.05	26.96	42
5th Bridge (E)	12	12.09	2.25	13.22	145.09	26.96	42
M11	210	8.63	0.23	46.01	1811.27	47.84	72
E11	165	10.25	0.26	48.47	1691.32	43.29	72
6th Bridge (M)	30	13.76	10.4	33.65	412.73	31.17	33
6th Bridge (E)	28	13.27	1.11	29.90	371.57	31.17	33
M12	359	8.73	0.56	38.58	3135.68	193.15	233
E12	280	10.91	0.71	31.69	3055.92	199.21	233
Totals: M	2127	–	–	–	17498.18	1431.42	1761
E	1790	–	–	–	18343.99	1432.75	1761
Averages:M	149.57	8.82	0.87	23.51	1249.87	97.17	125.79
E	127.71	10.56	1.06	22.61	1310.29	102.34	125.79

*This timing is greater than the calculated time because ritardandos, accelerandos, and fermatas were not taken into account in my data.

EXAMPLE 9: STATISTICAL RESULTS OF MOVEMENT ANALYSIS



EXAMPLE 10: MEASURED SPEED OF MOTION IN *LICHTER—WASSER*.



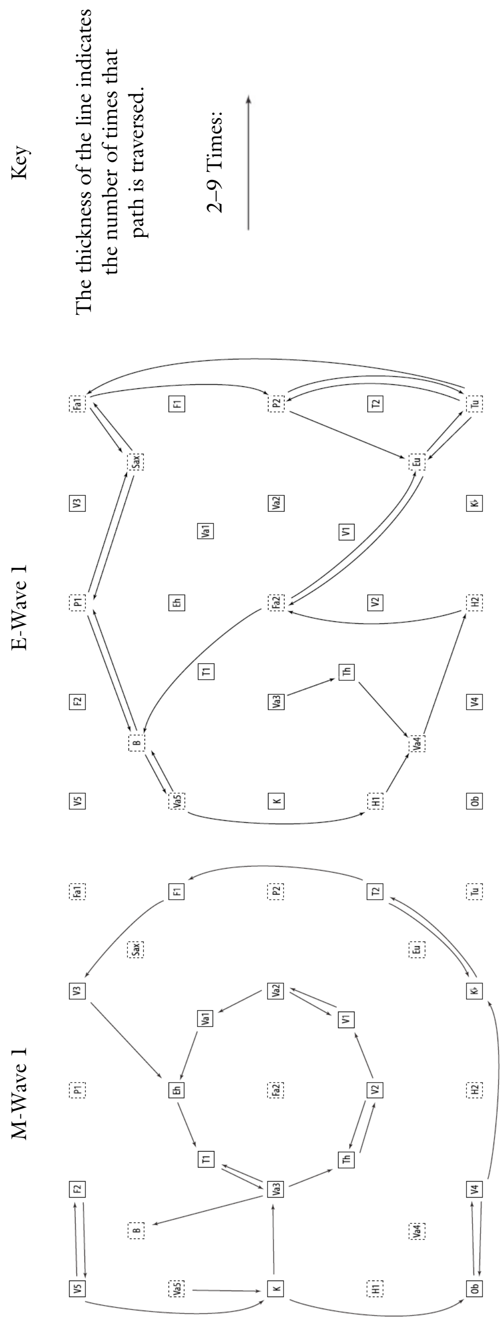
EXAMPLE 10 (CONT.)

indicate that the rate of spatial motion does indeed increase throughout the work, starting at approximately nine meters per second in wave 1 (which is equivalent to about twenty miles per hour) and peaking at about forty-eight meters per second in wave 11 (approximately 108 miles per hour).[39] The overall increase in the rate of movement serves to validate Stockhausen's general claim, but the fact that the rate occasionally *decreases* from one wave to the next suggests that perhaps Stockhausen was not so simple-minded as to compose in such an obviously linear way. Even though the primarily concern in the present analysis is not the perception of spatial motion, is it possible to speculate how listeners might follow such extremely rapid moves around the performance space?

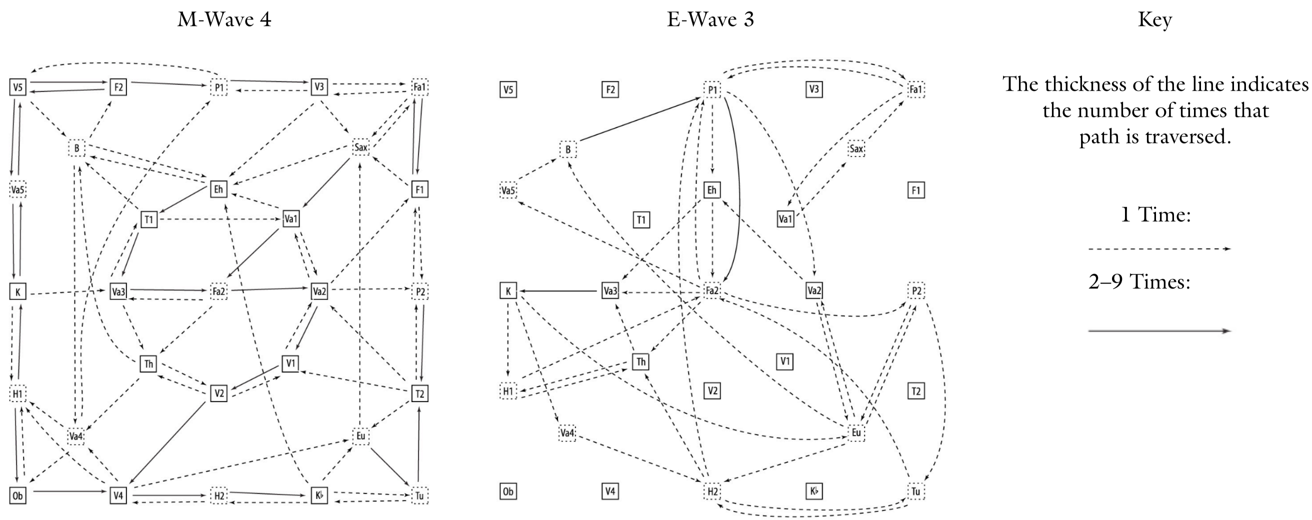
Stockhausen carefully crafted the spatial motion in *Lichter—Wasser* in order to make it easier to follow. As we can see in Example 11, the first pair of waves (M-wave 1 and E-wave 1, hereafter referred to as "M1"and "E1") generally follow the basic cycle of spatial movement. These basic cycles are the same as those shown in Example 6. As the work progresses, Stockhausen varied the patterns of spatial movement so that by M4 and E3, significantly different patterns emerge. More complex and diffuse patterns of M4 and E3 can be seen in Example 12. But as the speed of spatial movement increases still further, to a point where following intricate twists and turns around the hall might become difficult or impossible, Stockhausen returned to patterns reminiscent of the basic cycles. The wave sections with the most rapid overall movement, M11 and E11, return to the simple, repetitive shapes of the basic cycles.[40] These structures are shown in Example 13. So, as the spatial motion becomes more rapid, the shapes created in space become more repetitive and predictable.

Do the timbres of the instruments involved in *Lichter—Wasser* aid the listener in following the apparent spatial motion around the hall? A clue may lie in Stockhausen's remarks concerning electronic music composition. Regarding his eight-channel spatial electronic piece *Oktophonie*, Stockhausen once wrote that:

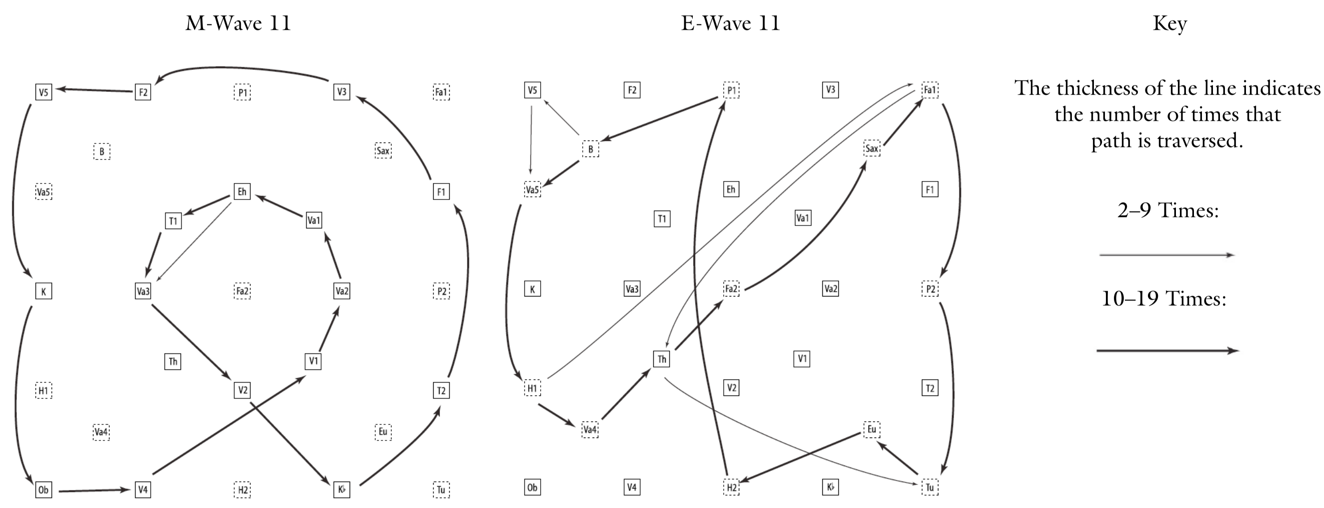
> In order to be able to hear . . . movements—especially simultaneously—the musical **rhythm** has to be drastically slowed down; the **pitch** changes must take place less often and only in smaller steps or with glissandi, so they can be followed; the composition of **dynamics** serves the audibility of the individual layers—i.e. dependent on the timbres of the layers and the speed of their movements; the **timbre** composition primarily serves the elucidation of these movements. (Stockhausen 1994, xxviii)



EXAMPLE 11: SPATIAL STRUCTURE IN M1 AND E1



EXAMPLE 12: THIRD BLOCK OF WAVES (ONLY M4 AND E3) SHOWING INCREASINGLY COMPLEX AND DIFFUSE SPATIAL STRUCTURE.

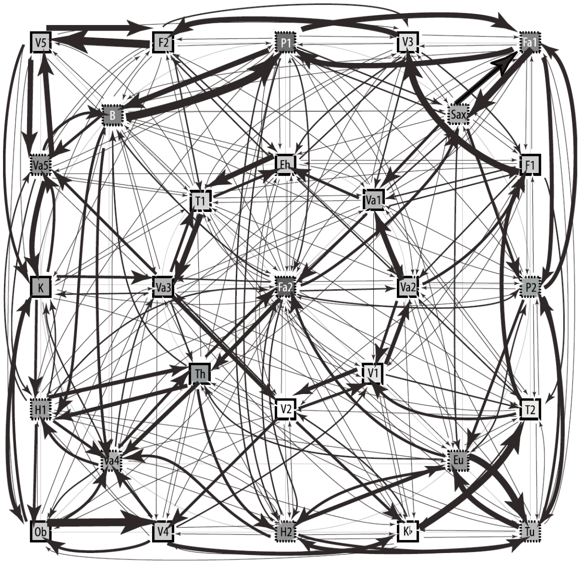


EXAMPLE 13: SIXTH BLOCK OF WAVES (M11 AND E11) SHOWING A RETURN TO SIMPLE, REPETITIVE MOTIONS SIMILAR TO THE BASIC CYCLES

Stockhausen's remark that the timbre composition in *Oktophonie* serves to elucidate the spatial dimension of the work might also be applied to *Lichter—Wasser*, since the timbres of the melodies are constantly in flux. Another composer, Henry Brant, mentioned (also in regard to electronic music), that "If what emerges from any given speaker is not sufficiently differentiated from the musical content assigned to other speakers, in relation to texture, range, and tone quality, more bewilderment and a sense of 'wasted' space result" (Brant 1978, 237). It would seem from the observations of Brant and Stockhausen, both of whom had considerable experience composing and listening to spatialized music, that changes in timbre might actually help to differentiate parts of the space around listeners.

Other factors which come to light only through analysis contribute even more to the differentiation of space around listeners in *Lichter—Wasser*. Example 14 illuminates a subtle aspect of the work. The shading of the instruments indicates how often each one is used during the course of all twelve wave sections. The darkly shaded instruments are used more often, whereas the lightly shaded ones are put into action less frequently. As can be seen from the example, instruments in the southeast quadrant are used less often than instruments in other quadrants. While there is clearly some regional variation, the difference in the use of individual instruments is also significant. The most infrequently used instrument—violin 2—is heard only 102 times, but the most often heard instrument—bassoon 2—is put to work 171 times. The average usage of each quadrant is shown at the bottom of Example 14. These data show that instruments in quadrant 1 are used an average of 141 times, whereas those in quadrant 3 are only used an average of 129 times. This disparity of nearly nine percent may may help to calculate a pay scale for the musicians, but it also allows us to see that some areas of space in *Lichter—Wasser* are indeed individualized.[41]

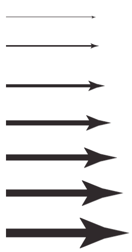
Counting the number of occurrences of each instrument in *Lichter—Wasser* makes it possible to see that the instrument **th**, or tenor horn, is of particular importance. As an instrument in the Michael orchestra, it should—like all Michael instruments—be used primarily to pass Michael melodic material along. However, this is not the case. Measurements indicate that even though it is used 124 times in the Eve orchestra, it is only used twenty-one times in the Michael orchestra. Further analysis reveals that one other instrument, **va5** (viola 5), exhibits the opposite characteristics. Although this instrument is technically a part of the Eve orchestra, it is used ninety-three times in the Michael orchestra but only fifty-seven times for Eve material. The way these two instruments are "shared" can be interpreted as an



Key 1: Shading for each instrument based on the number of times it is used.

110 120 130 140 150 160 170 180

Key 2: Thickness of the line indicates the number of times that path is traversed.



2–9 times:
10–19 times:
20–29 times:
30–39 times:
40–49 times:
50–59 times:
60–69 times:

Quadrant 1: v5, f2, p1, b, va5, eh, t1, k, va3, fa2	Average number of times used: 141
Quadrant 2: p1, v3, fa1, sax, eh, f1, va1, fa2, va2, p2	Average: 138 times
Quadrant 3: fa2, va2, p2, v1, v2, t2, eu, h2, kb, tu	Average: 129 times
Quadrant 4: k, va3, fa2, th, h1, v2, va4, ob, v4, h2	Average: 137 times

EXAMPLE 14: COMMON INSTRUMENTAL MOVES IN *LICHTER—WASSER*

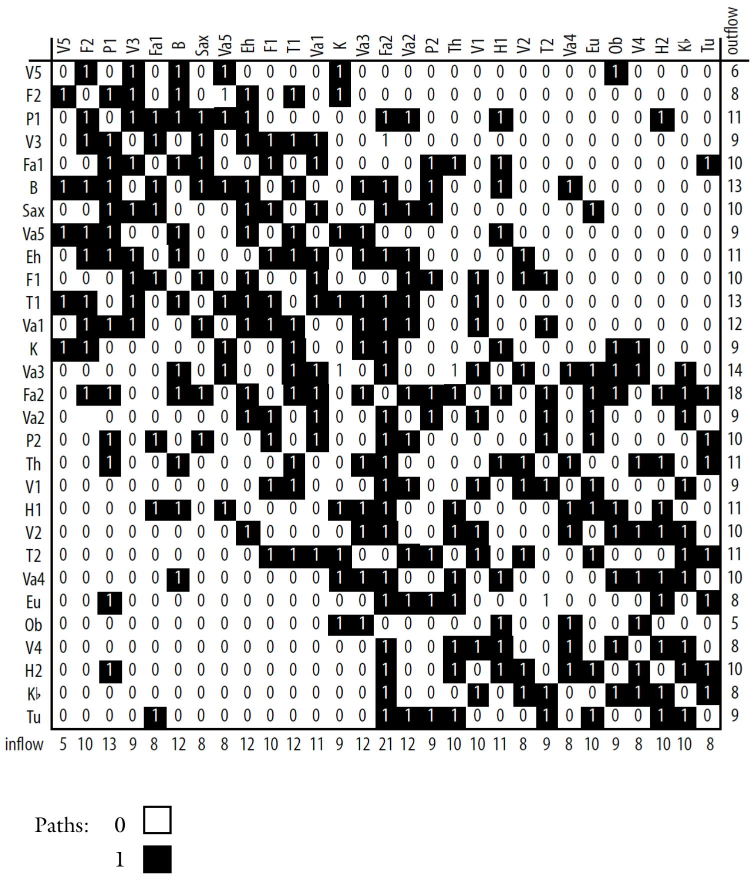
expression of the "mystical union" between the work's two protagonists.[42] This is a subtle aspect of the program in *Lichter—Wasser* which would be impossible to know without having analyzed the spatial motion.

Along with information relating to the number of times each instrumentalist is used, the overall, out-of-time structure of movement that Stockhausen created in space is also shown in Example 14. Thick lines and arrows indicate that a move occurs very frequently, while thin lines indicate that it happens rarely.[43] Example 14 allows us to make one important connection with contour theory. The idea of "pruning" a complex musical contour to yield a more simple one was developed by Robert Morris (1993). It is possible to see how a much simpler implementation of pruning than Morris's can apply to spatialized music by imagining the graph in Example 14 with successive layers stripped away. Given an arbitrary range of occurrences of a particular spatial event, that event is eliminated at the next level. The simpler pruning procedure in spatial music makes it possible to see deeper spatial structures at work, similar to the way pruning a melodic contour can be used to see a more global musical shape of a given passage. If thinner lines are pruned away in Example 14, it becomes apparent that a great deal of motion cycles around the center in two concentric circles.[44] Moreover, the diagonal from the northeast corner to the southwest corner is traversed much more frequently than the other diagonal. That these familiar forms come to light upon analysis may suggest that simpler, intuitive Gestalt structures play an important role in Stockhausen's spatial language.

–3–

In the previous section, several important aspects of spatialization in *Lichter—Wasser* were analyzed using relative simple analytical techniques such as counting and averaging. Now, it is possible to apply some of the more sophisticated techniques of analysis related to graph theory, starting with the adjacency matrices **A1**, **A2**, and **A3**. These matrices are shown in Examples 15, 16, and 17. Importantly, these matrices only indicate whether an instrument has a connection to another instrument or not; they *do not* indicate how many times an instrument passes along melodic elements to another instrument.

Compared to their physical arrangement in space, the arrangement of instrumentalists along the top and left sides of the **A1** matrix (and all adjacency matrices we will examine) is from left to right, and top to bottom. This arrangement in the matrix allows us to align each instrumentalist with itself on the diagonal from northwest to southeast. An advantage to this way of organizing the matrices is that each quadrant in the matrix generally corresponds to that quadrant's relationships with itself and the other three quadrants in the physical space of the piece. Thus, it is possible to see in a general way from the shading not just how well each instrument is connected to each other instrument, but also how each quadrant is connected to each other quadrant. On the **A1** matrix, the last column and row are labeled "outflow" and "inflow." These figures sum the connections shown in



	V5	F2	P1	V3	Fa1	B	Sax	Va5	Eh	F1	T1	Va1	K	Va3	Fa2	Va2	P2	Th	V1	H1	V2	T2	Va4	Eu	Ob	V4	H2	K♭	Tu	outflow
V5	0	1	0	1	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	6
F2	1	0	1	1	0	1	0	1	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8
P1	0	1	0	1	1	1	1	1	1	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	1	0	0	11
V3	0	1	1	0	1	0	1	0	1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9
Fa1	0	0	1	1	0	1	1	0	0	1	0	1	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	1	10
B	1	1	1	0	1	0	1	1	1	0	1	0	0	1	1	0	1	0	0	1	0	0	1	0	0	0	0	0	0	13
Sax	0	0	1	1	1	0	0	0	1	1	0	1	0	0	1	1	1	0	0	0	0	0	0	1	0	0	0	0	0	10
Va5	1	1	1	0	0	1	0	0	1	0	1	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	9
Eh	0	1	1	1	0	1	0	0	0	1	1	1	0	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	11
F1	0	0	0	1	1	0	1	0	1	0	0	1	0	0	0	1	1	0	1	0	1	1	0	0	0	0	0	0	0	10
T1	1	1	0	1	0	1	0	1	1	1	0	1	1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	13
Va1	0	1	1	1	0	0	1	0	1	1	1	0	0	1	1	1	0	0	1	0	0	1	0	0	0	0	0	0	0	12
K	1	1	0	0	0	0	0	1	0	0	1	0	0	1	1	0	0	0	0	1	0	0	0	0	1	1	0	0	0	9
Va3	0	0	0	0	0	1	0	1	0	0	1	1	1	0	1	0	0	1	1	0	1	0	1	1	1	1	0	1	0	14
Fa2	0	1	1	0	0	1	1	0	1	0	1	1	0	1	0	1	1	1	0	1	0	1	0	1	1	0	1	1	1	18
Va2	0		0	0	0	0	0	0	1	1	0	1	0	0	1	0	1	0	1	0	0	1	0	1	0	0	0	1	0	9
P2	0	0	1	0	1	0	1	0	0	1	0	1	0	0	1	1	0	0	0	0	0	1	0	1	0	0	0	0	1	10
Th	0	0	1	0	0	1	0	0	0	0	1	0	0	1	1	0	0	0	0	1	1	0	1	0	0	1	1	0	1	11
V1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1	0	0	1	0	1	1	0	1	0	0	0	1	0	9
H1	0	0	0	0	1	1	0	1	0	0	0	0	1	1	1	0	0	1	0	0	0	0	1	1	1	0	1	0	0	11
V2	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	1	1	0	0	0	1	0	1	1	1	1	0	10
T2	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	1	1	0	1	0	1	0	0	1	0	0	0	1	1	11
Va4	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	0	0	0	1	1	1	1	0	10
Eu	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	1	0	0	0	0	1	0	1	8
Ob	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	5
V4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	0	0	1	0	1	0	1	1	0	8
H2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1	1	0	1	1	0	1	0	1	1	10
K♭	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	1	0	0	1	1	1	0	1	8
Tu	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	1	0	1	0	0	1	1	0	9
inflow	5	10	13	9	8	12	8	8	12	10	12	11	9	12	21	12	9	10	10	11	8	9	8	10	9	8	10	10	8	

Paths: 0 □
1 ■

EXAMPLE 15: THE A1 MATRIX FOR *LICHTER—WASSER*

each row and column to give a measure of how many edges lead into each node, and how many lead out. Later, outflow and inflow will be used to measure centrality.

It is already apparent from the **A1** matrix that there is a large number of possible paths between instruments that are placed near

	V5	F2	P1	V3	Fa1	B	Sax	Va5	Eh	F1	T1	Va1	K	Va3	Fa2	Va2	P2	Th	V1	H1	V2	T2	Va4	Eu	Ob	V4	H2	K♭	Tu
V5	4	4	4	1	2	2	2	3	4	1	5	1	3	4	3	0	1	0	0	4	0	0	2	0	1	2	0	0	0
F2	4	8	4	4	3	5	3	5	5	3	5	3	3	5	6	3	1	0	1	4	1	0	1	0	2	1	1	0	0
P1	3	5	9	4	4	6	4	3	7	5	6	6	3	5	7	3	5	4	1	5	2	2	3	5	2	1	2	3	3
V3	2	5	6	8	3	6	5	3	7	5	4	6	2	4	5	7	4	2	2	3	2	3	0	2	1	0	2	1	2
Fa1	1	4	6	4	8	3	6	3	6	4	4	4	1	4	9	6	4	2	2	3	2	4	3	4	1	1	4	1	2
B	3	6	7	7	4	11	4	6	6	5	5	7	7	6	8	6	3	5	2	5	2	2	2	5	5	2	4	3	3
Sax	0	5	7	5	4	4	7	1	6	6	4	7	0	3	7	7	5	3	3	3	2	6	0	3	2	0	3	2	4
Va5	4	6	3	5	3	7	2	8	4	2	5	3	5	5	7	3	1	2	2	3	2	0	3	2	4	2	2	1	0
Eh	3	6	5	5	4	5	6	5	10	4	6	6	3	5	8	5	4	3	6	3	2	4	3	3	3	2	3	4	1
F1	0	3	6	4	3	2	4	0	5	9	5	7	1	3	8	6	4	2	5	1	3	4	1	5	1	1	1	4	3
T1	4	8	7	5	3	6	5	5	8	5	10	6	4	6	8	5	4	2	5	4	4	5	2	4	4	2	1	4	1
Va1	2	5	5	6	4	6	4	4	8	7	7	9	4	3	8	8	5	2	6	2	5	4	1	6	2	1	2	5	2
K	3	4	3	3	1	7	1	5	4	1	4	3	7	5	4	2	1	4	3	4	1	1	4	3	5	2	3	3	1
Va3	4	6	6	2	1	5	3	3	6	3	7	2	4	10	11	5	3	5	6	8	3	5	5	2	7	6	7	5	4
Fa2	3	5	9	6	6	7	4	6	7	7	7	7	6	7	15	9	6	5	6	5	6	6	6	8	4	5	6	5	6
Va2	0	3	5	3	2	2	4	0	3	5	5	5	1	3	6	8	4	2	5	1	5	7	0	4	3	1	3	3	5
P2	0	3	5	5	4	3	5	1	6	5	3	6	1	2	6	8	8	4	4	3	2	6	0	5	2	0	4	4	4
Th	2	4	3	2	4	6	3	5	5	1	3	3	4	6	10	4	3	8	4	6	2	2	6	5	6	4	7	7	2
V1	1	2	2	2	1	2	2	1	5	4	3	5	2	3	6	6	5	3	7	1	4	6	1	4	4	2	4	5	4
H1	3	4	7	1	1	6	3	3	3	1	6	3	4	7	7	2	4	6	1	9	3	2	5	3	5	6	4	4	5
V2	0	2	4	1	0	5	1	1	1	2	5	3	3	5	8	3	1	5	4	6	6	3	5	4	5	6	5	6	4
T2	2	3	3	3	3	1	3	2	5	5	3	4	1	4	10	7	4	3	7	1	3	8	1	4	4	3	4	4	3
Va4	2	3	4	0	2	4	2	4	2	0	5	2	3	6	8	1	2	5	3	7	4	2	7	4	6	6	5	4	4
Eu	0	2	4	1	3	3	3	1	3	3	3	4	2	3	6	5	4	3	2	5	3	4	3	6	1	3	4	5	5
Ob	1	1	0	0	1	3	0	3	0	0	2	1	3	3	5	0	0	4	2	3	1	0	3	2	5	3	3	3	0
V4	0	1	3	0	1	4	1	1	1	1	3	1	3	5	6	2	1	4	2	5	4	3	4	4	4	5	5	4	4
H2	0	2	3	1	3	5	2	2	3	0	2	1	2	5	9	4	3	7	3	5	2	4	4	3	7	4	10	5	4
K♭	0	1	2	0	1	1	1	0	2	2	3	2	2	3	5	4	3	5	4	4	3	3	4	5	3	3	4	7	3
Tu	0	1	6	1	1	3	3	0	2	4	3	5	1	2	6	4	5	4	3	4	4	5	2	5	3	3	4	4	8

Paths: 0
1–3
4–6
7–9
10–12
13–15

EXAMPLE 16: THE A2 MATRIX FOR *LICHTER—WASSER*

each other in physical space. The lack of connections among instruments far from each other can be seen by the large number of zeros (white space) in the northeast and southwest corners of the graph.

Examining the **A2** matrix, it can be seen that the northwest to southeast diagonal are filled with the greatest number of potential

	V5	F2	P1	V3	Fa1	B	Sax	Va5	Eh	F1	T1	Va1	K	Va3	Fa2	Va2	P2	Th	V1	H1	V2	T2	Va4	Eu	Ob	V4	H2	K♭	Tu
V5	17	30	23	27	15	39	15	30	26	16	25	23	27	28	35	21	10	17	13	22	9	6	13	14	22	10	15	11	6
F2	26	44	44	38	24	50	29	38	50	28	45	38	34	40	50	31	23	22	22	31	15	17	19	24	25	13	17	19	11
P1	23	54	62	49	41	61	46	42	60	42	53	52	34	54	87	60	39	37	34	44	29	38	27	37	37	24	42	29	32
V3	20	49	58	43	39	44	43	32	62	50	56	55	25	41	76	52	38	23	35	30	27	35	20	38	21	14	23	28	22
Fa1	15	41	61	43	32	53	42	26	55	48	46	59	27	41	72	55	46	40	32	41	27	36	20	44	29	18	33	37	36
B	35	67	76	47	45	63	52	50	74	46	73	56	40	69	102	57	48	41	40	61	33	42	40	43	43	34	42	37	34
Sax	14	41	60	46	41	43	45	26	62	54	50	60	24	39	78	65	48	29	40	31	32	44	20	48	20	15	31	35	33
Va5	31	51	52	32	23	53	31	38	53	28	54	37	38	53	62	31	26	29	25	46	18	21	27	25	34	24	25	26	18
Eh	25	56	68	50	33	61	41	36	63	56	66	61	38	57	90	63	40	34	44	44	39	41	26	45	36	26	34	39	33
F1	11	38	48	42	33	37	43	21	57	47	44	55	18	36	73	65	44	29	47	26	33	51	14	39	27	16	34	35	31
T1	33	63	65	56	37	67	44	49	74	56	68	64	48	63	95	68	42	35	52	44	36	42	29	44	42	27	38	39	31
Va1	26	59	69	51	37	51	50	34	74	61	65	64	30	58	97	71	49	34	55	39	37	56	23	44	37	24	37	40	36
K	27	43	43	24	21	45	23	37	38	23	50	27	35	52	68	29	21	28	25	45	24	21	31	25	36	32	29	26	20
Va3	25	50	56	38	34	72	33	50	56	40	65	56	55	66	104	58	39	57	52	60	47	41	51	62	58	46	55	62	39
Fa2	31	73	88	55	50	81	63	49	88	64	83	80	49	81	133	88	68	70	65	75	49	69	45	70	66	45	70	72	57
Va2	11	30	41	32	29	30	32	20	51	46	40	50	22	33	72	58	43	30	47	25	31	45	20	46	24	20	32	43	31
P2	11	36	60	37	38	37	42	20	52	55	47	58	20	37	82	63	46	30	40	30	35	50	20	50	23	19	33	36	41
Th	22	43	65	28	26	62	32	34	46	33	59	43	40	64	94	44	37	52	36	63	40	39	45	47	52	46	53	47	46
V1	10	29	42	25	22	29	27	16	39	42	44	43	22	37	72	53	35	30	45	28	37	46	23	44	29	26	32	43	34
H1	23	43	48	31	36	61	30	46	46	26	47	37	44	60	92	45	30	50	35	54	28	30	46	44	51	38	55	42	31
V2	16	31	37	18	24	42	24	30	41	22	41	30	32	53	83	40	30	49	43	47	31	35	43	40	49	40	54	49	32
T2	10	34	49	31	23	40	33	18	47	47	51	55	28	38	71	59	44	36	48	35	40	48	23	51	33	24	35	50	40
Va4	19	34	43	22	23	55	22	34	37	21	42	30	42	57	80	38	27	53	33	54	27	28	43	40	51	40	53	46	32
Eu	11	27	43	25	28	36	28	22	38	32	34	38	22	37	74	47	38	41	35	33	27	40	25	41	36	24	43	38	35
Ob	12	18	23	6	6	26	10	16	16	6	25	11	21	33	36	12	11	24	15	33	15	13	25	16	27	25	24	20	18
V4	12	22	30	12	19	34	17	24	27	15	31	24	26	40	65	30	26	43	30	40	25	25	36	34	40	33	43	40	28
H2	13	28	51	16	23	45	25	24	33	26	44	36	31	49	78	37	34	49	30	56	37	32	45	49	39	42	46	49	44
K♭	7	17	30	14	16	30	16	16	24	23	28	27	21	34	65	35	25	35	34	30	30	35	26	34	36	31	41	36	33
Tu	8	27	40	25	34	31	31	20	40	32	33	36	20	37	76	55	39	39	37	33	28	44	27	44	31	24	46	41	34

Paths: 0–22

23–45

46–68

69–91

92–114

120–143

EXAMPLE 17: THE A3 MATRIX FOR *LICHTER—WASSER*

paths, given the connections Stockhausen made between instruments. This illustrates the fact that there are many possible ways to get from a given instrument and back to it in two moves. Shapes of this sort are called *cycles* since they end where they began. The possibility of so many cycles opens up an important question: did Stockhausen avail himself of these shapes in the spatial design? An algorithm which found cycles of degree 2 turned up only sixty in all of the twelve wave sections of *Lichter—Wasser*. The majority of these occur only once: the most common cycle, which occurs four times, is (**sax–fa1–sax**).

As we analyze the **A3** matrix, a different structure becomes apparent in the potential spatial moves of *Lichter—Wasser* for paths of degree 3 involving a maximum of four instrumentalists. Here, it appears that that the diagonal in the matrix from northwest to southeast is relatively sparse in connective possibilities. It is possible to hypothesize that given the small number of measured cycles of degree two, the general trend in spatial motion in *Lichter—Wasser* is to move away from the starting point, instead of doubling back on itself. In other words, if Stockhausen moves his melody from a first instrument to a second, and then onward to a third, the third instrument will probably no longer be adjacent to the first, and therefore it will be less likely that the spatial motion will return to the first instrument, forming a closed cycle of degree 3. This tendency is illustrated in the sparseness of potential cycles in the northwest–to–southeast diagonal. However, the cycle-finding algorithm found slightly *more* cycles of degree 3 than of degree 2—altogether, seventy. This is still a small number, given the total number of moves in the piece. No cycle of degree 3 occurs more than twice.

Clearly, the **A2** and **A3** adjacency matrices are good for predicting what possible paths may occur, but they do not tell us what actual paths do happen. It would be wrong then to expect that the *actual* number of paths indicated in the matrices correspond to the *potential* number of paths. Still, adjacency matrices do at least two things well: first, they give an exact idea of how to get from one place in space to another in a piece of spatial music, given the connections the composer chose to make. Second, they are a rough measure of how well each instrument is connected to each other in piece's deeper spatial structure. Further analysis of adjacency matrices, cycles, and other spatial shapes would be a fruitful way to continue this analysis.

–4–

Centrality is an important concept that has been developed over the last fifty years in the published literature, and was alluded to in the test violin piece. What does centrality mean in a graph? One convenient way to imagine a graph with a very strong central node or vertex is to consider the arrangement of spokes on a bicycle tire. This graph is usually called a *star* graph. In this structure, there are nodes at the ends of all the spokes, and one central node in the middle. This is the central node in the graph because it is connected to all the other nodes by exactly one edge. In measures of centrality, the central node would have the highest numerical value. In graphs that are more complex than our bicycle spoke structure, there are many ways of calculating the centrality index of a node.[45] One of the simplest of these measures is called *degree centrality*.[46] The degree centrality (C_D) of a vertex v is given as

$$C_D(v) = \text{degree}(v) / n - 1$$

where n is the total number of edges in the graph, and the *degree* of v is the number of edges incident to the node being analyzed. This value can be determined both for the *inflow* of a node, and the *outflow*. It is possible to calculate the degree centrality of all twenty-nine instruments in the graph of *Lichter—Wasser*, both regarding their inflow and outflow.

The first analysis of centrality in *Lichter—Wasser* is based on the synchronic (out-of-time) graph which takes into account all the moves that happen more than once in the piece. Later, each wave will be examined by itself to see how the central node changes over time, presenting a diachronic (in-time) view of the spatial motion. The synchronic view is based on the numbers given in the last column and last row of the **A1** matrix. The results of this calculation are shown in Example 18. In Example 18, the degree centrality of both inflow and outflow are shown; the two figures are then averaged, for an overall measure of centrality.

The data shown in Example 18 allow several conclusions to be drawn about the structure of spatial motion in *Lichter—Wasser*. First, it should come as no surprise that the instrument in the geographic center of the graph, bassoon 2 (**fa2**) has the highest degree centrality. Several other instruments, however, score high on this measure. Chief among them are **b** (bass clarinet), **t1** (trumpet 1), and **va3** (viola 3). These instruments are all located in quadrant 1, an indication that this

Instrument	Inflow	Degree Centrality	Outflow	Degree Centrality	Overall Degree Centrality
v5	5	0.0171	6	0.0205	0.019
f2	10	0.0342	8	0.0274	0.031
p1	13	0.0445	11	0.0377	0.041
v3	9	0.0308	9	0.0308	0.031
fa1	8	0.0274	10	0.0342	0.031
b	12	0.0411	13	0.0445	0.043
sax	8	0.0274	10	0.0342	0.031
va5	8	0.0274	9	0.0308	0.029
eh	12	0.0411	11	0.0377	0.039
f1	10	0.0342	10	0.0342	0.034
t1	12	0.0411	13	0.0445	0.043
va1	11	0.0377	12	0.0411	0.039
k	9	0.0308	9	0.0308	0.031
va3	12	0.0411	14	0.0479	0.045
fa2	21	0.0719	18	0.0616	0.068
va2	12	0.0411	9	0.0308	0.036
p2	9	0.0308	10	0.0342	0.033
th	10	0.0342	11	0.0377	0.036
v1	10	0.0342	9	0.0308	0.033
h1	11	0.0377	11	0.0377	0.038
v2	8	0.0274	10	0.0342	0.031
t2	9	0.0308	11	0.0377	0.034
va4	8	0.0274	10	0.0342	0.031
eu	10	0.0342	8	0.0274	0.031
ob	9	0.0308	5	0.0171	0.024
v4	8	0.0274	8	0.0274	0.027
h2	10	0.0342	10	0.0342	0.034
kb	10	0.0342	8	0.0274	0.031
tu	8	0.0274	9	0.0308	0.029

Degree Centrality of Quadrants:

Quadrant 1: v5, f2, p1, b, va5, eh, t1, k, va3, fa2
Average overall degree centrality: 0.039

Quadrant 2: p1, v3, fa1, sax, eh, f1, va1, fa2, va2, p2
Average: 0.038

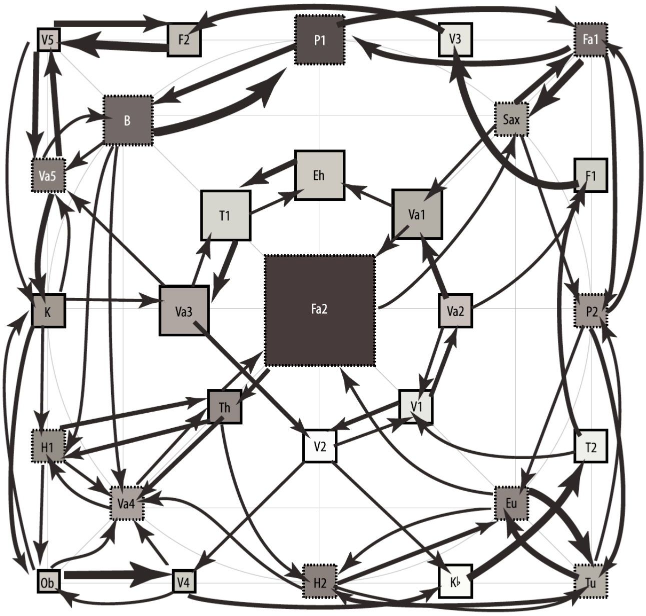
Quadrant 3: fa2, va2, p2, v1, v2, t2, eu, h2, kb, tu
Average: 0.036

Quadrant 4: k, va3, fa2, th, h1, v2, va4, ob, v4, h2
Average: 0.037

EXAMPLE 18: DEGREE CENTRALITY OF EACH INSTRUMENT IN *LICHTER—WASSER* quadrant is not only used more often than the others, but also that it is more central to the graph as a whole. Example 18 also shows the average degree centrality for the four quadrants of space. By only a small margin, instruments in quadrant 1 are more central to the graph than those in others. As one might expect from their physical location in the graph, instruments at the corners (**v5**, **fa1**, **ob** and **tu**) score relatively low, and therefore can be considered the least connected nodes to the graph. Example 19 visually illustrates the connected-ness of each instrument to the graph based on the degree centrality measure. The size of each instrument's box is proportional to its graph centrality; its shading continues, as before, to indicate the number of times it is used. Example 19 is also "pruned" to show only the more common moves among instrumentalists.

Aside from their physical distance to the central hub, what other factors contribute to the isolation of the corners of space from the rest of the graph? Since modularity is not in force in this piece, instruments towards the edges such as violin 5 and oboe simply have fewer places to send their melodic flow, and therefore function more like conduits.[47] The number of places that corner instruments can send their material is further limited by the tendency of melodic material to stay within one of the two orchestras; this creates further restrictions on where an instrument can pass its melody to.[48] Third, Stockhausen's reluctance to make large spatial moves further contributes to the relative isolation of the corner instruments. If large leaps were common, corner instruments would have more places to direct their melodic material.[49]

Although it may seem convenient to classify instruments either as conduits or hubs, such a taxonomy may sometimes be problematic. Consider the use of the tenor horn once again, as it is shown in Example 19. It is apparent from the "pruned" graph that this instrument serves as the most common conduit between the outer circle and the inner circle both in terms of inflow and outflow. Further emphasizing this role, the tenor horn has these strong connections with *three* instruments: **va4** (viola 4), **fa2** (bassoon 2), and **h1** (horn 1). But despite its function as an important conduit, the tenor horn also makes many connections with a variety of other instruments, due to its location in the space. By our measure of centrality, it scores 0.036—a value that indicates it is slightly above average (0.034) in its connectedness to the graph as a whole. We may conclude that the tenor horn is a kind of hybrid instrument which exhibits some characteristics of both hub and conduit.



Degree Centrality: 0.018–0.028

0.029–0.038

0.039–0.048

0.049–0.058

0.059–0.068

EXAMPLE 19: MAP OF DEGREE OF CENTRALITY IN *LICHTER—WASSER*

Now, the structure of spatial motion can be analyzed in a different way. Instead of a synchronic analysis, it is possible to arrive at a diachronic view by investigating the change of centrality over time, from wave to wave. Since Michael and Eve waves do not always begin and end together, it is necessary to analyze centricity of each Michael and Eve wave individually, rather than pairing E- and M-waves together. For this more detailed analysis, *weighted* adjacency matrices were used, which include data on exactly how many times an instrument passes melodic material to another instrument.

Some of the results of this analysis are shown in Example 20. Each wave has one, or occasionally two instruments that are central to it. In the case of the Michael waves, this center changes for each wave, and with only two exceptions (**f1** and **va1**—both of which are included in the table because they tie other instruments) there is a different instrument that functions as center for each wave. The Eve waves behave somewhat differently, since the instrument in the middle of the physical space—bassoon 2—functions as center four times. The great majority of central nodes are in the upper two quadrants—only four of them lie to the "south" of the central horizontal axis **k–p2**, further demonstrating that the most dense activity occurs in these two regions.

	Michael Orchestra		Eve Orchestra	
	Central Instrument	Degree Centrality	Central Instrument	Degree Centrality
Wave 1	va3	0.076	b	0.100
Wave 2	f1	0.105	fa1	0.067
Wave 3	k	0.054	p1	0.116
Wave 4	v4	0.052	fa2	0.095
Wave 5	t1	0.075	fa2	0.113
Wave 6	v3	0.078	fa2	0.104
Wave 7	Eh, f1	0.060	p1	0.082
Wave 8	va5	0.083	p1	0.080
Wave 9	va1	0.112	sax	0.106
Wave 10	fa2	0.116	tu	0.096
Wave 11	Va1, va2	0.067	va5	0.082
Wave 12	ob	0.078	fa2	0.104

EXAMPLE 20: CENTRAL NODE(S) IN EACH WAVE OF *LICHTER—WASSER*

The way in which centers of spatial activity move around the space, wave-by-wave, can be interpreted as a higher-level structural thread running through the piece. However, this higher-level structure is not necessarily reflected in lower-level structures. Analysis indicates that the instrument which serves as a center in M1 (**va3**) *never* moves to **fl**, the central instrument in M2; furthermore, **b**, the center of E1 only goes to **fa1** (center of E2), twice (**b** is much more likely to move to **p1**). From this analysis, it is possible to conclude that constructing spatial movement in a hierarchical way is not a part of the compositional strategy or the spatial structure of *Lichter—Wasser*.[50]

–5–

What about the idea of symmetry and transformation in *Lichter—Wasser*? The fixed location of the twenty-nine instruments (which is symmetrical along four axes) as well as the arrangement of instrumentalists within the two orchestras (which is symmetrical around the horizontal axis **k** (clarinet)—**p2** (trombone 2)) might lead one to hypothesize that creating symmetrical motions along one of these axes might be a compositional strategy. However, the motions that were measured in this analysis generally do not exhibit this symmetry. For example, if there were a larger number of spatial cycles, it might be easier to relate spatial shapes in the piece through mathematical transformations. Despite the cycles we measured above, the spatial shapes that are formed in *Lichter—Wasser* are more often quite elongated, elaborate and asymmetrical. The shapes that *are* actually related to each other through operations of rotation, reflection, or translation often contain melodic material which is not strongly connected musically; for example, there may be lengthy rests between notes in a spatial shape, or dramatic changes in register (Miller 2009, 134–38). This makes segmenting the musical surface into transformationally related shapes a highly questionable endeavor.

Are there abstract similarities between the pitch and spatial domains in *Lichter—Wasser*? In other words, considering the group of T_n and T_nI operations on pitch, is there an isomorphism in operations that transform one spatial location to another? Given the way that Stockhausen composed his spatial motion, this too seems unlikely. While operations of rotation and reflection around the center of the space in concentric circles could easily form dihedral or cyclic groups, motion from one ring to another would be problematic, since it would not always involve one-to-one operations. Limiting spatial motion only to concentric rings around the center axis would put constraints on the variety and shape of instrumental motion in the work.[51] Getting to the very important instrument **fa2** at the center of the space would be very problematic under these circumstances.

Could the space in *Lichter—Wasser* be thought of as modular? Indications from box diagrams, as mentioned above, show that melodic material never moves off the edge of the space and back onto it. Although the perception of spatial motives is not considered in this article from any formal perspective, it seems unlikely that the internal coherence of such a motive could be maintained when it is transposed or translated off the edge of the space—presumably involving some kind of modularity in the process.[52] Multiplication operations, which could dilate or contract a spatial shape, may help to express the relationship between the two concentric circles in play in *Lichter—Wasser*, but they are not always one-to-one, and their use might not allow for a great deal of spatial variety. This is not to say that transformations relating shapes in spatial music can never be part of a mathematical group structure; in fact, the test piece with nine violinists seems a much more promising candidate since there are far simpler shapes that can be formed with only nine musicians.

Early sketches for *Lichter—Wasser* show that Stockhausen actually considered a number of symmetrical spatial shapes of increasing complexity, but he changed his mind during the actual composition (Stockhausen 2001, 21). It seems as though Stockhausen mostly favored *asymmetrical* shapes in *Lichter—Wasser* which are not clearly related by conventional notions of mathematical transformation. Only a few transformational relationships among spatial shapes can be found in the work; furthermore, the similarity of these shapes as motives is potentially undermined by the fact that their corresponding pitch and rhythmic underpinnings are not closely coordinated. It is difficult to comprehend a group of operations that relate spatial shapes together to form a musically convincing structure analogous to transposition and inversion in pitch in this work. It would therefore seem reasonable to conclude that these two domains operate mostly according to different principles in *Lichter—Wasser*.[53]

–6–

Measurements of the spatial motion in *Lichter—Wasser* have unveiled a surprising level of detail which has, up until now, gone unnoticed.

Simply by measuring spatial motion, it is possible to conclude that the movement in space follows a general process of speeding up from wave 1 to wave 12, essentially verifying the composer's claims. Despite very rapid spatial movement around listeners towards the end of the piece, some coherence is maintained through a strategy of using simpler spatial patterns and recognizable melodies. By measuring the number of times each instrumentalist is used, it was possible to observe a subtle way in which two instruments, tenor horn and viola 5, are shared between orchestras, elegantly expressing the work's programmatic idea of mystical union. Different quadrants of the space were then analyzed separately, based on how often instrumentalists are used. Finally it was possible to show, by representing the links between instrumentalists as a mathematical graph, how individual musicians play different functions or roles in the work as conduits, hubs, or hybrids, based partly on their position in space.

One lens which has been fruitfully applied to many analyses of Stockhausen's music—and has, notably, not been mentioned here—is serialism. Indeed, serialism was originally the context in which Stockhausen thought of spatialization, and there is considerable evidence showing that he attempted to construct spatial motion according to serial principles in *Kontakte* (1958–60) and *Licht—Bilder* (2002–2003).[54] However, non-serial techniques of organization often figured just as largely into Stockhausen's compositional practice as more strict serial processes.[55] From the preceding analysis of *Lichter—Wasser*, it is clear that the organizational aspects relating to space are not determined by serial rows, matrices or other deterministic principles, but rather are improvised. While serial thinking casts its shadow over the way the *temporal duration* of each wave section is determined—in other words, the "mold" into which the apparent spatial moves are set—it does not appear to influence the structures we have measured and analyzed in space.

Although these methods were useful for analyzing the spatial aspect of *Lichter—Wasser*, different approaches may be crafted for other spatial works since composers generally have a very highly individual approach to the domain. This is particularly true for electronic music, which has not figured into the present study in any significant way. Still, measuring the distances, speeds and densities of musical material in spatial music, and thinking of it in terms of a mathematical graph structure—permitting the use of the adjacency matrix and the degree centrality measure—have yielded significant results when applied to *Lichter—Wasser*. With only slight modifications, such tools could be effectively applied to other spatialized music, or to create interesting spatial structures in entirely new works.

Notes

1. Evidence of space playing a role in music goes back to archaeological findings in ancient Greek amphitheaters (Lewcock 2001). Spatialization in Medieval music often centered around traditions of chant singing and the use of hocket. Willaert pioneered the technique of *cori spezzati* in the mid-sixteenth century and Zarlino defined the term in *Le istitutioni harmoniche*. Richard Rastall suggested that for works printed in a "Tafelmusik" layout, a spatial structure is implied (Rastall 1997). Composers such as Dowland, Shepherd, Gombert, Tye, and Byrd all printed music in this format. Berlioz's *Requiem* is an excellent example of the dramatic use of spatialized brass ensembles. The sonic effect of distance that Mahler used at the beginning of his first symphony is more overtly exploited in the fifth movement of the second symphony (Adorno 1991, 4). Many other creative uses of spatialization can be found in music before 1950.

2. Some of the innovative spatial ideas from the 1950s and 60s were already prefigured by Robert Beyer much earlier (Beyer 1928).

3. It is important to draw a distinction between studies of the *location* of sounds in space, and studies of the *compositional techniques by which* a composer decides on how to spatialize those sounds. This article engages the latter question, leaving the former problem to others. Regarding questions of perception, and the dispersion of sounds in space, the reader is referred to the large-scale study by Blauert 2001 as only one possible starting point.

4. Various approaches to devising a method or system for analyzing compositional choices in spatialized music can be found in Maria Harley 1994, Nauck 1997, Overholt 2006, Hofmann 2008, and Solomon 2010. Brant 1978 offers a view of the issues involved in spatialization from the composer's perspective. Interest has extended considerably beyond compositional decisions; cross-disciplinary studies have enlivened the dialog between composers, architects, neuroscientists, and many others; in particular, Blesser/Salter 2007 make a notable contribution in this direction. Acousticians—particularly those interested in architectural acoustics—have taken a great interest in spatialization, as well as corporations such as Dolby laboratories which have utilized spatialization in public and home theater systems.

5. In their lecture at the 2010 Stockhausen Courses in Kürten, the Freiburg audio engineers Joachim Haas and Gregorio Karman—who assisted Stockhausen in his realization of the work *Cosmic Pulses* (2007)—discussed the way in which small "bumps" in frequency were intended to mimic the Doppler effect in that work.
6. Although Leonard Meyer believed that the idea of treating different musical parameters independently was "at least open to serious question," it seems clear that when spatialization is concerned, we must take several parameters into account if we are to develop a cognitive model of localization (Meyer 1994, 252).
7. According to Morris, "most musical systems quantize the pitch-continuum into a finite number of pitches ordered from low to high" (Morris 1987, 23). We could take the view that spatialization is simply *not* a "musical system"—a conclusion that the present author is not particularly inclined to make—or, we could say that unlike "most" musical systems, spatialized music operates in a space which has a finite or infinite set of points ("locations") in a one-, two-, or three-dimensional field. Many of Morris's ingenious and useful techniques of structuring music could then be modified to operate in this world. Of course, it should be noted that the title of Morris's book ("Composition With Pitch Classes") probably ought not to raise readers' expectations in the direction of offering insight to musical spatialization, and is instead targeted towards exploring the most common medium through which musical communication occurs—namely, pitch. Spatialized music poses other problems in the context of established traditions of music theory. For example, most kinds of spatial movement in two or three dimensions would probably violate both conditions A and B of Lewin's formulation of the GIS (Lewin 2007, 26) since a potentially infinite number of points *t* could lie at an interval *i* from a starting point *s*.
8. Trochimczyk's ingenious classification schemes for spatialized works provide an excellent overview of the possibilities of arranging musicians or loudspeakers in space (Trochimczyk 2001). Conen surveys Stockhausen's basic spatialization schemes up to the mid-1970s (Conen 1991, 26–33). Imke Misch provides indispensable insight to Stockhausen's directions for spatialization in his scores, but does not build an analytical system or methodology for the spatial structures she identifies (Misch 1999a). Nauck's 1997 classification of motion types in Stockhausen's *Gruppen*—based on spatial and timbral attributes—

is the most successful accomplishment in the analysis of spatial music to date.

9. Konrad Boehmer briefly investigated the links between modern and Renaissance spatialization practice in his 1961 essay "Raum-Formen." Boehmer believed that Stockhausen's pronouncements on spatialization (especially in his 1959 *die Reihe* article "Musik im Raum") were overblown, and were based on a misunderstanding of earlier (i.e., Renaissance) practice. According to Boehmer's analysis, spatial thinking in the music of the 1950s and early 60s arose out of a "direct formal need" [*unmittelbaren formalen Bedürfnis*]. In other words, the *only* difference between the older impulse for the use of space and the modern one was that spatialization arouse *out of* the techniques used in the Renaissance, whereas the modern forms *demanded* that spatialization be used. (Boehmer 2009, 56–57).

10. It would be useful to develop methods for analyzing electronic spatial music; however, these works pose their own special problems and are explicitly not considered here. For more on possible methodological approaches, see Miller 2009, 48–50.

11. The eight-channel mixdown version that Stockhausen made of *Lichter—Wasser* is not a good substitute, since it is only an approximation of the experience of twenty-nine live musicians scattered around and between the audience.

12. Boehmer remarked at length on the highly individualized audience experience of spatial music (Boehmer 2009, 58–59), as well as Brant 1978, 224.

13. In the situation just proposed, the melody jumps disjointly from one musician to another. Although some studies suggest that space is not the only force at work in grouping structures (Deutsch 1973, 1974), Bregman concludes that space can be a grouping mechanism (Bregman 1990, 74). Inspired by Bregman's work and from Gestalt psychology, Solomon 2010 extends Bregman's ideas in his formulation of the "spatial gesture." Using the analogy of a person's footsteps, Solomon suggests that even when melodies or sounds jump disjointly, they can—under certain circumstances—be perceived as one continuous gesture.

14. Similar tendencies can be found in the way that scale degrees function in tonal music. In many different musical repertoires, scale degree 7 has a strong tendency to move to scale degree 1; it

tends also to move to scale degree 5. Measuring spatial movement could allow us to hear similarities between the way pitch functions and the way that location functions in space. "Key profiles"—such as those developed by Kessler and Krumhansl (1982) and others—have generated a great deal of interest in exploring listeners' tonal expectations.

15. Graphs have, of course, appeared countless times in the music theoretic literature, and it is worthwhile citing a few of the many instances briefly. Weber's "Tabelle der Tonartenverwandtschaften" from his *Versuch* (see Bernstein 2002, 786) is an early example of a graph model of tonal relations. The oft-cited *Tonnetz* of Riemann and others is another important graph structure in music theory, as is Schoenberg's "Chart of the Regions." Douthett and Steinbach (1998) have, in the best neo-Riemannian tradition, illustrated the structure of several subtle tonal structures through the use of graphs, to which they assign imaginative appellations. Outside the Riemannian universe, Allen Forte's K/Kh graphs show inclusion relationships as a graph structure (Forte 1973). Robert Morris (1987) has elegantly shown how partial orderings of PCs may be illustrated in lattice graphs (198–220ff.). Lewin's classic analysis of Stockhausen's *Klavierstück III* uses two different graphs to portray the relationships among pentachords he identified in that work (Lewin 1993).

16. More formally: "A graph is an ordered pair $(V(G), E(G))$ consisting of a set $V(G)$ of *vertices* and a set $E(G)$, disjoint from $V(G)$, of *edges*, together with an *incidence function* c_G that associates with each edge of *G* an ordered pair of (not necessarily distinct) vertices of *G*" (Bondy and Murty 2008, 2).

17. Including this information could be done; the graph would then be a *weighted graph*.

18. Given nodes *a* and *b* in a graph, a *weighted adjacency matrix* contains the number of times that the directed path from *a* to *b* is traversed. We will use such a matrix later in the analysis.

19. It follows that an $\mathbf{A}^n$ matrix would tell which paths are available between any two instrumentalists involving *n* moves or a maximum of $n + 1$ intermediate musicians, inclusive.

20. But, as the matrix shows, there is no way to do the reverse: that is, get from violin 5 to violin 3 in two steps. This illustrates the

important distinction between *inflow* and *outflow*, which we will explore in more detail later.

21. Stockhausen said to Gisela Nauck, "instrumental and electronic spatial composition have been my artistic mandate since 1951" (Nauck 1997, 173). The importance of spatialization can be observed in his last major work, *Cosmic Pulses* (2007), where Stockhausen made exactly 1,928 (8 × 241) notated moves in space —a hidden reference to the year of his birth.

22. The characters Michael and Eve which appear in Stockhausen's seven operas, have great significance for Stockhausen's theology and world-view. Lucifer, the third character, does not appear in *Lichter—Wasser* (though he does make several appearances in the *Sunday* opera). These three characters—or, perhaps better put, "essences"—stem partly from Christian beliefs and also from Stockhausen's reading of the *Urantia Book*. Wager (1998), Peters (2003), Bandur (2005), and Ulrich (2006) have investigated these connections more closely.

23. *Lichter—Wasser* Lectures, Kürten, 2001, day 1. It is not unreasonable to suggest that Stockhausen might have been thinking of the following words in the *Urantia Book* 55:0.0: "The age of light and life is the final evolutionary attainment of a world of time and space."

24. The reason for the curious misalignment of wave sections lies in the deeper formal structure of the work and its scheme of proportional temporal expansion of melodic material (Stockhausen 2001, 25).

25. It is tempting to think of these "basic cycles" as a fundamental ordering of the instrumentalists—somewhat akin to a tone row.

26. Of course, at the time Stockhausen composed *Lichter—Wasser*, Pluto was still considered a planet. In a personal conversation that took place in 2006, Stockhausen mentioned that he felt it was a mistake to demote Pluto: "it will only confuse people."

27. The fact that the motions of planets themselves did not relate directly to the motions in the music was made absolutely clear in Stockhausen's response to a questioner on day 4 of his lectures on *Lichter—Wasser* in Kürten, 2001.

28. The general strategy Stockhausen seems to have employed was to have the vocalists move around the instrumental ensemble in ways

that are dissimilar and complementary to the melodic motion passed around in the orchestras. An exhaustive study of the vocalists' movements can be found in Miller 2009, 108–109, and 127–29.

29. These "box diagrams" as I call them, are a standard feature in Stockhausen's scores since *Carré* (1959–60).

30. See Toop 2006, 104 for a color reproduction of Stockhausen's 1977 sketch of the nuclear formulas. Also, the superformula (translated into English) can be found on page 199 of the same book.

31. The sketches for and development of the superformula can be found in Stockhausen 1989, 147–60. The "Akzidenzien" are listed and defined in detail on 160.

32. The most important exception, of course, is the first act of the Tuesday opera, *Jahreslauf*, whose composition predates LICHT and is not associated with the superformula.

33. The most important of these modifications is that intervals from the Eve-formula are gradually "blended" into the Michael-formula through the twelve wave sections. However, the reverse does not occur. For a more extended discussion of this process, see Stockhausen 2001, 44–45, and Miller 2009, 100 and 260–61. Another modification is that the formulas are dilated either 1×, 2×, 3×, or 4× with respect to their fundamental rhythmic values. The logic behind this compositional choice has to do with the way that Michael and Eve waves do not always line up, and can be gleaned in Stockhausen 2001, 25.

34. Stockhausen often fixed the contour of the superformula melodies in such a way as to make them more recognizable. The detail which he assigned melodic shape in these melodies can be plainly understood from an interview Stockhausen had with Rudolf Frisius (Stockhausen 1998, 13–28).

35. Stockhausen provided listeners with one additional way to disambiguate Michael and Eve melodies in *Lichter—Wasser*. On a stand next to each musician is a lighted candle, which is either in a green glass bowl or a blue one. These colors indicate membership in one of the two ensembles: green for the Eve orchestra, blue for the Michael orchestra.

36. An objection might be raised concerning the dependency of our measurements on the size of the hall in which the work is performed. However, there are clear limits to the size of the hall possible for performing *Lichter—Wasser*. If the hall is too small, there will be no room for the audience to sit; if it is too large, the musicians will not be able precisely to coordinate the spatial handoffs over larger distances. Small variations in layout will, of course, cause the measurements to change. But much as in a twelve-tone analysis, changes in hall size affect only the measurements themselves, *not* the relationships among different measurements. It would be somewhat like calling a row-form starting on the pitch class F P_0 instead of the more customary P_5: the labels of the rows are different, but the relationships among rows remain the same.

37. The numerous ritardandi and accellarandi in the score are not taken into account in the measurements.

38. Occasionally, spatial motion splits into two or three paths, though a single path is by far the norm. The occasional splits usually set up a kind of spatial canon which nevertheless still allows one to follow a single path of motion through the passage. These events are rare during the wave sections.

39. Stockhausen was not the only composer to delight in rapid spatial motion: Xenakis used similar compositional techniques as well. In his 1992 interview with Maria Harley, Xenakis said, "when the sound moves along with the speed of a walking human it is not interesting enough. If you could ask the players to run and play at the same time, that would be more interesting." (Maria Harley 1994, 299). Xenakis continued by remarking that there were probably not many musicians who would want to run around while performing. His own personal solution to rapid spatial motion can be observed in *Terretektorh*, where, using methods similar to the ones we are developing for Stockhausen's music, the author was able to analyze some motions peaking at 253 meters per second (566 miles per hour)—an extremely fast speed! (Miller 2009, 322–23).

40. The twelfth M- and E-waves actually contain the most rapid measurable movement, but because of the numerous pauses in activity, their calculated *average* speed is actually slower than in M11 and E11.

41. There is also a correlation between the register of the instrument and the number of times it is used. By informally grouping instruments into categories of high, medium and low range, those in the first category were used an average of 116 times; those in the medium range were used an average of 138 times; instruments in the lowest category were used an average of 152 times. Many lower-pitched instruments, such as the euphonium and tuba, are placed in quadrants 3 and 4. As this rough analysis indicates, Stockhausen tended to use instruments of lower register more often in *Lichter—Wasser*.

42. Although there are many theatrical elements of "mystical union" in the action of *Lichter—Wasser*—most notably the elaborate entrance procession—the most obvious way in which musical material is shared between Michael- and Eve- orchestras can be heard in the gradual transformations of the Michael pitch material through the twelve wave sections. As mentioned above, the Michael melodies slowly take on intervals that characterize the Eve melody (curiously, the reverse is not the case. See Stockhausen 2001, 44–45). Although a kind of "mystical union" plainly occurs in the pitch domain, the one that occurs in the spatial domain is not at all self-evident, and was never acknowledged by Stockhausen during his lifetime.

43. Moves that only occur once are not shown, since they would clutter the already dense graph and potentially introduce unwanted noise into the calculations. Therefore it could be said that the analysis represents the deep foreground, middleground and background events in space.

44. The idea of two concentric rotating structures may very well have been a reflection on Stockhausen's part of the way that the supposed "belts of dark gravity bodies" rotate in the Havona universe. See the *Urantia Book*, 14:1.8.

45. These measures have a wide variety of uses, from calculating the likelihood that a computer in a network might become infected with a virus, to finding the best hub for an industrial supply chain. Centrality is also an important concept in evaluating the way individuals behave in a social network, and how highly to rank a web page in a search engine.

46. This measure, along with several others, was developed in Freeman 1977 and Freeman 1979.

47. If the physical space in *Lichter—Wasser* were modular, it would be likely that continuous spatial melodies would go off the edge of the space, returning on the other side of it as if nothing had happened. Clearly this is not the case in the work, as can be seen by a cursory glance at the many box diagrams and from the previous examples above.

48. Though—as mentioned before—**th** and **va5** operate mostly in the "wrong" orchestra.

49. As can be seen in Example 9, the average length of a move in the Michael orchestra is 8.82 meters, while the average move in the Eve orchestra (whose members are already slightly more widely spaced than the M players) is 10.56 meters (recall that the hall is approximately 27 × 30 meters in size.)

50. This could not contrast more with Stockhausen's treatment of many other aspects of the work—especially the form, pitch, and rhythm—which are derived hierarchically from the Superformula itself.

51. Some arrangements of musicians in spatialized music may have more overt connections to the structure of the pitch domain. For example, arranging the choirs in a single circle in Tallis's famous 40-part motet *Spem in alium* can result in similar transformational group structures to pitch. See Miller 2009, 159–63.

52. Wayne Slawson's work on timbre provides the best possible model for developing mathematical group structures in spatialized music (Slawson 1985).

53. While this is contrary to Stockhausen's early serialist manifesto (Stockhausen 1961, "Music in Space"), it is more in keeping with his later, more experience-based approach to spatialization (Stockhausen 1978, 360–424, "Vier Kriterien der Elektronischen Musik").

54. The most lucid exposition of Stockhausen's idea of serial spatial organization in *Kontakte* is found in Stockhausen 2009, 232–45. As far as the present author knows, Stockhausen made no further attempts to organize spatial motion according to serial principles after *Kontakte* except for the physical movements of the four soloists in *Licht-Bilder* (2002/3). M. J. Grant, however, emphasized the close ties between serialism and spatialization. Spatialization enhances the "impossibility of preparing oneself mentally for the sounds which will occur" in electronic music. This

inability to predict what might happen is—according to her—an "aesthetic ideal of serial music" (Grant 2001, 99).

55. The most detailed study of serial and non-serial processes in Stockhausen's music can be found in Kohl 1981; in works like *Gruppen*, entire sections ("insertions/Einschübe") are improvisational (Misch 1999b, 142–52).

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